Alternative Methods to Test for Positive Assortative Mating

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Abstract

We propose easy econometric methods to test for positive assortative mating. Compared to existing methods such as by Siow (2015), our procedure avoids the cost associated with imposing positive assortative mating restrictions, and it allows for more general tests. The test does not require numerical optimization, and the test statistics can be calculated easily. We compare the power of the tests using simulation. The empirical application suggests that the null hypothesis of no assortative mating is rejected for selected years of CPS data that range from 1962 to 2019.

1 Introduction

This paper proposes easy methods to test for positive assortative mating. A number of empirical studies on sorting in the marriage market have investigated the intensity of assortative mating and how the assortativeness changes over time. Only a handful of methods based on econometric tests have been developed for testing positive assortative mating.

A strand of literature on marriage matching market is initiated by Choo and Siow (2006b). They propose a marriage matching function that is based on the transferable utility matching framework with random preferences.[1,2] The critical assumptions of Choo and Siow (2006b) are (a) additive separability of joint surplus, (b) large marriage market, and (c) extreme value distribution of the shock (Galichon and Salanié, 2021). Choo and Siow (2006a) applied the marriage matching function to quantify the effect of the legalization of abortion on gains to marriage.

[1] See Chiappori and Salanié (2016) for the survey of the empirical marriage matching models under different assumptions: transferable utility, non-transferable utility, and imperfectly transferable utility models.

[2] See Fox (2010), Fox (2018), and Fox, Yang, and Hsu (2018) for an alternative method employing a maximum score approach based on the rank-order property associated with sorting. See Suen and Lui (1999) for a correlation-based test.

Based on the transferable utility framework, papers in the literature extend Choo and Siow (2006b) in different directions. Extending the structural model of Chiappori, Iyigun, and Weiss (2009), Chiappori, Salanié, and Weiss (2017) build a marriage matching model with parental investment and embed it into the Choo and Siow (2006b) marriage matching function. They measure the changes in assortative mating by estimating the returns to education in the US marriage market. Mourifié and Siow (2021) incorporate peer effects into the marriage matching function of Choo and Siow (2006b). Dupuy and Galichon (2014) suggest how to amend the Choo and Siow (2006b) model for continuous attributes based on the continuous choice framework developed by Dagsvik (1994) and Dagsvik (2000). Galichon and Salanié (2021) show that the extreme value assumption of Choo and Siow (2006b) can be relaxed.

Few papers develop econometric tests for positive assortative mating. Siow (2015) proposes a test for positive assortative mating based on maximum likelihood estimation. The test statistics are based on the theoretical framework of Choo and Siow (2006b), and the test procedure requires numerical optimization with different restrictions. Our method does not require numerical optimization and can be easily calculated. The likelihood ratio test of Siow (2015) mainly uses two restrictions to define the null hypothesis: one imposes the positive restriction of the entire cells of types, and the other focuses on the preference for own types (TP2[3] and DP2[4]). It is unclear which of the restrictions should be used in testing for positive assortative mating.[5] It is not clear which of the restrictions in Siow (2015) are proper to assess positive assortative mating. For example, the log-likelihood with the TP2 restriction is rejected against the unrestricted likelihood, but the log-likelihood with the DP2 restriction is not rejected. Siow (2015) states that the rejection of the TP2 likelihood is driven by the couples whose completed education is distant. The extreme matches account for less than 0.2% of marriages. The results suggest that TP2 might be a too strong restriction for testing for positive assortative mating. Our method requires only the unrestricted estimates. Also, it exploits all elements of type-cells but is not as strict as imposing TP2, allowing for more general tests.

We describe our method in Section 2. In Section 3, we provide empirical examples using Current Population Survey samples between 1962 and 2019. We conclude in Section 4.

[3] TP2 is when all local log odds ratios are positive of order 2 usually denoted as "$\prec_2$." For two random variables A, B, we say that $A \prec_2 B$ iff

$$\int_{-\infty}^{x} [\Pr(B > t) - \Pr(A > t)]\, dt \geq 0$$

$\forall x$ and $\exists x$: the inequality in the equation is positive. See, for example, Shaked and Shanthikumar (2007).

[4] DP2 is when all log odds ratios on the diagonal are positive of order 2.

[5] Chiappori, Salanié and Weiss (2017) also propose a test based on their structural model that incorporates returns to parental investment. Their method requires using minimum distance of the moment conditions that come from their structural model. The main purpose of this test is to examine the changes in the intensities of educational homogamy.

2 Alternative Tests

Let n_{ij} be the number of observations where a type-i male, $i = 1, 2, .., I$ is matched with a type-j female, $j = 1, 2, .., J$, and define $N = \sum_{ij} n_{ij}$ as the total number of sampled marriages. Let p_{ij} be the probability that a randomly sampled marriage is of type ij. Siow (2015) states that the unrestricted log likelihood function for this problem is

$$\begin{aligned} L &= \sum_{i,j} n_{ij} \log p_{ij} \\ st\ 1 &= \sum_{ij} p_{ij}. \end{aligned}$$

The unrestricted MLE of p is $\widehat{p}^u = \left\{\widehat{p}_{ij}^u\right\}$ with $\widehat{p}_{ij}^u = n_{ij}/N$. Siow (2015) and others cited by Siow (2015) argue that the way to test for positive assortative mating is to also estimate $\widehat{p}$ under the restrictions imposed by positive assortative mating and then perform a likelihood ratio test. The null hypothesis in Siow (2015) is a region with positive measure, and the alternative hypothesis is a region. This is a quite uncommon problem in the literature.

One example receiving some attention is the unit root hypothesis in a time series. Almost all of the literature on testing for a unit root assumes that the null hypothesis is nonstationarity. A strong reason for making this assumption instead of stationarity being the null hypothesis is that, when nonstationarity is the null hypothesis, it can be represented by a single $AR(1)$ parameter equalling 1. Thus, the situation is turned into a standard case where the null hypothesis corresponds to a point. However, from an economic point of view, the null hypothesis should be stationarity in that the model behaves better when there is stationarity (we would like to not reject the null hypothesis and then rely on the nice properties of stationary processes. Kwiatkowski et al. (1992), Leybourne and McCabe (1994), and Xiao (2001) all use a null hypothesis of stationarity by constructing a time series model with a random walk where the random walk is the only source of nonstationarity. The null hypothesis is then that the variance of the random walk error is equal to zero.

A second example, much closer to our problem, is to test whether the Slutsky substitution matrix is negative semidefinite. Gill and Lewbel (1992) choose a null hypothesis of semidefiniteness (which has positive measure and show how to construct a consistent test statistic. Gill and Lewbel (1992) state that the asymptotics of this problem are very similar to that in Kodde and Palm (1986) where the null has positive measure.[6] [Steve: Please add more explanations regarding the comparison of ours with Siow. Siow's null hypothesis is a region, which makes the testing a lot harder. It

[6] Unfortunately, Cragg and Donald (1996, 1997) show that the asymptotics for a related rank condition are incorrect, and neither paper addresses the correctness of the argument of Gill and Lewbel (1992) about testing for semidefiniteness.

also makes it difficult to generate the distribution of their test statistics. Ours is simpler that we just wanna compare the distance from a point. We talked about some graphical illustrations as well.] An easier approach is to construct a "pseudo-Wald test," requiring only the unrestricted estimates.[7][Jay: add short discussion of other tests for assortative mating in literature.] This avoids the cost associated with imposing the positive assortative mating restrictions, and it allows for more general tests.

The condition for positive assortative mating is[8]

$$R_{ij} = (\log p_{ij} + \log p_{i+1,j+1}) - (\log p_{i+1,j} + \log p_{i,j+1}) > 0 \ \forall ij, \tag{1}$$

and the proposed test statistic for testing for positive assortative mating is

$$T = \sum_{i,j} \left(\log \widehat{p}^u_{ij} + \log \widehat{p}^u_{i+1,j+1}\right) - \left(\log \widehat{p}^u_{i+1,j} + \log \widehat{p}^u_{i,j+1}\right)$$

where a superscript u indicates "unrestricted."

One might worry that the distribution of T under either the null or alternative would be difficult to evaluate. However, the same is true of Siow (2015) because the restrictions are inequalities instead of equality restrictions (Kudo, 1962; Gourieroux, Holly, and Monfort, 1982; Kodde and Palm, 1986). Siow (2015) uses parametric bootstrapping to simulate critical values. The same can be done for T. Under positive assortative mating, $T > 0$, and, with total positive assortative mating (TP2 in Siow, 2015), $T \to \infty$ because $p_{i+1,j} p_{i,j+1} = 0$. Under an alternative specification, random sorting, where $\log p_{ij} = \log x_i + \log y_j$[Jay: is this ours or Siow's?], using the definition of R_{ij} in equation (1),

$$R_{ij} = (\log x_i + \log y_j + \log x_{i+1} + \log y_{j+1}) - (\log x_{i+1} + \log y_j + \log x_i + \log y_{j+1}) = 0.$$

One should be able to simulate a critical value $\widehat{c}$ such that

$$\Pr\left[T > \widehat{c} \mid R_{ij} = 0 \ \forall ij\right] = \delta.$$

Note that one could test the hypothesis that $R_{ij} = 0 \ \forall ij$ with power against the alternative of (positive or negative) assortative mating as a Wald test with

$$\sum_{i,j} \frac{\widehat{R}^2_{ij}}{s^2_{ij}} \sim \chi^2_K$$

[7] It's not clear which should be considered the restricted model and which the alternative.

[8] Note that $R_{ij} > 0 \ \forall ij \Rightarrow$

$$\left(\log p_{ij} + \log p_{i+k,j+k}\right) > \left(\log p_{i+k,j} + \log p_{i,k+2}\right) \ \forall k.$$

where

$$\widehat{R}_{ij}^2 = \left(\log \widehat{p}_{ij}^u + \log \widehat{p}_{i+1,j+1}^u\right) - \left(\log \widehat{p}_{i+1,j}^u + \log \widehat{p}_{i,j+1}^u\right),$$

$$\begin{aligned} s_{ij}^2 &= Var\left[\left(\log \widehat{p}_{ij}^u + \log \widehat{p}_{i+1,j+1}^u\right) - \left(\log \widehat{p}_{i+1,j}^u + \log \widehat{p}_{i,j+1}^u\right)\right] \\ &\approx \frac{(1-p_{ij})}{Np_{ij}} + \frac{(1-p_{i+1j+1})}{Np_{i+1j+1}} + \frac{(1-p_{i+1,j})}{Np_{i+1,j}} + \frac{(1-p_{i,j+1})}{Np_{i,j+1}}, \end{aligned}$$

using a first-order Taylor series approximation, and

$$K = (I-1)(J-1).$$

Also note that the MLE of p under the restriction of random mating is $\widehat{p}^r = \left\{\widehat{p}_{ij}^r\right\}$ where

$$\widehat{p}_{ij}^r = \widehat{x}_i \widehat{y}_j, \quad \widehat{x}_i = \frac{\sum_j n_{ij}}{N}, \quad \widehat{y}_j = \frac{\sum_i n_{ij}}{N}.$$

Thus, one could test the null hypothesis of random mating against a general alternative with a likelihood ratio (LR) test,

$$\begin{aligned} T_{LR} &= 2\left[\sum_{i,j} n_{ij} \log \widehat{p}_{ij}^u - \sum_{i,j} n_{ij} \log \widehat{p}_{ij}^r\right] \\ &= 2\left[\sum_{i,j} n_{ij} \log \frac{\widehat{p}_{ij}^u}{\widehat{p}_{ij}^r}\right] \\ &= 2\left[\sum_{i,j} n_{ij} \log \frac{\widehat{p}_{ij}^u}{\left(\sum_k \widehat{p}_{ik}^u\right)\left(\sum_k \widehat{p}_{kj}^u\right)}\right] \sim \chi_K^2 \end{aligned}$$

(which requires only estimation of $\widehat{p}^u$) with

$$K = MF - 1$$

which is larger than the degrees of freedom in for the pseudo-Wald test. This difference exists because, for the pseudo-Wald test, one loses degrees of freedom at $m = M$ and $f = F$ which does not happen for the LR test.

Alternatively, one could construct a Lagrange Multiplier (LM) test.[Steve: Need beginning of sentence]

$$\begin{aligned} L &= \sum_{i,j} n_{ij} \log p_{ij} \\ \frac{\partial L}{\partial p_{ij}} &\mid_{p_{ij}=\widehat{p}_{ij}^r} = \frac{n_{ij}}{\widehat{p}_{ij}^r} \\ &= \frac{n_{ij}}{\frac{\sum_k n_{ik}}{N} \frac{\sum_k n_{kj}}{N}} \\ &= \frac{N\widehat{p}_{ij}^u}{\left(\sum_k \widehat{p}_{ik}^u\right)\left(\sum_k \widehat{p}_{kj}^u\right)} \end{aligned}$$

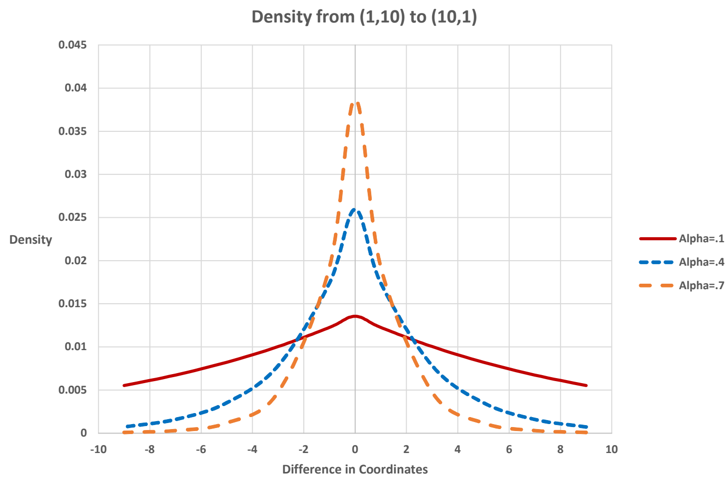

Figure 1: Density from (1,10) to (10,1)

implying that the LM will have the same form and the same distribution as the LR test.

[Steve: Explain that the simulated distns are more precise than the chi-sq distns]In order to simulate the distribution of our test statistics, we start with a model of assortative mating. Define a man as having an integer value of x_m between 1 and 10, and define a woman as having an integer value of x_w between 1 and 10. Assume that the density of matches in an economy is

$$f\left(x_m, x_w\right) = \frac{\exp\left\{-\alpha\left|x_m - x_w\right|\right\}}{\sum_{x'_m=1}^{10}\sum_{x'_w=1}^{10}\exp\left\{-\alpha\left|x'_m - x'_w\right|\right\}}.$$

Figure 1 shows the contour of $f\left(x_m, x_w\right)$ as $\left(x_m, x_w\right)$ moves from $(1, 10)$ to $(10, 1)$ for $\alpha = 0.1$, 0.4, and 0.7. The larger α, the more assortative mating there is. Figure 2 shows the distributions of the pseudo-Wald test for samples of 500, $1,000$, and $10,000$ when $\alpha = 0$ along with the χ^2_{81} distribution. The χ^2_{81} distribution exhibits less variance than the pseudo-Wald tests for samples of at least $1,000$, and it stochastically dominates the pseudo-Wald test distribution for a sample of 500. The 5% critical value for the χ^2_{81} distribution is off by a meaningful amount relative to the simulated pseudo-Wald test distributions. The 5% critical value for a test statistic with a χ^2_{81} distribution is 103.0, while for the pseudo-Wald tests with sample sizes 500, $1,000$, and $10,000$, they are respectively 99.0, 113.9, and 114.0.

Figure 3 shows the distributions of the LR test for samples of 500, $1,000$, and $10,000$ when $\alpha = 0$ along with the χ^2_{99} distribution. The χ^2_{99} distribution stochastically dominates the LR test distributions for all three sample sizes.

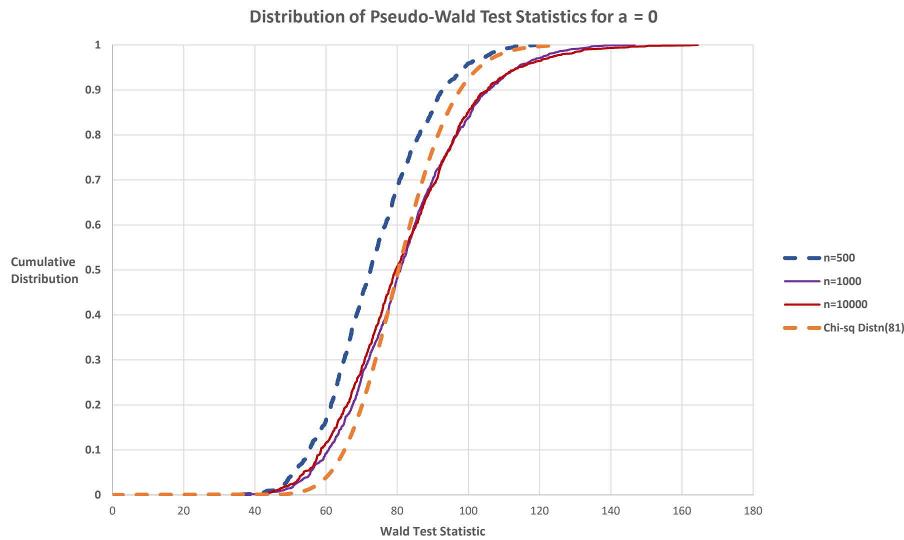

Figure 2: Distribution of Pseudo-Wald Test Statistic for $\alpha = 0$

The 5% critical value for the χ^2_{99} distribution is off by a meaningful amount relative to the simulated LR test distributions. The 5% critical value for a test statistic with a χ^2_{99} distribution is 123.2, while for the LR test distributions with sample sizes 500, 1,000, and 10,000, they are respectively 111.3, 107.3, and 107.4.

Figure 4 shows the power functions for both tests for different values of α using the 5% critical values under the null hypothesis of $\alpha = 0$. The pseudo-Wald test has weak power for sample sizes less than 10,000, and the LR test has very strong power. Part of the good performance of the LR test relative to the pseudo-Wald test is due to it having more degrees of freedom. Also, the LR test statistic distribution is dominated by the χ^2_{99} distribution while the pseudo-Wald test statistic is not dominated by the χ^2_{81} distribution; i.e., the LR test statistic has more power than implied by its χ^2_{99} asymptotic distribution while the pseudo-Wald test statistic does not have more power than its χ^2_{81} asymptotic distribution.

3 Empirical Example

Carroll, Kang, and Stern (2021) collect data from 18 Current Population Survey (CPS) samples between 1962 and 2019. For each sample, they estimate log wage rate equations using Heckman (1979), then compute log predicted wages using the inverse Mills' ratio $\phi(\cdot) / [1 - \Phi(\cdot)]$ for observed log wages and $-\phi(\cdot) / \Phi(\cdot)$ for unobserved log wages, and then measure the amount of assortative mating by log predicted wage rates among married couples. They also measure the

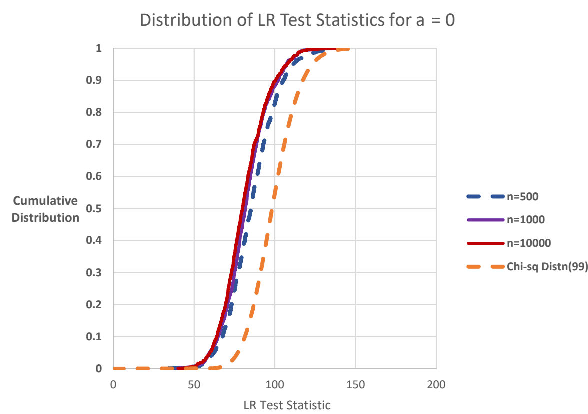

Figure 3: Distribution of LR Test Statistic for $\alpha = 0$

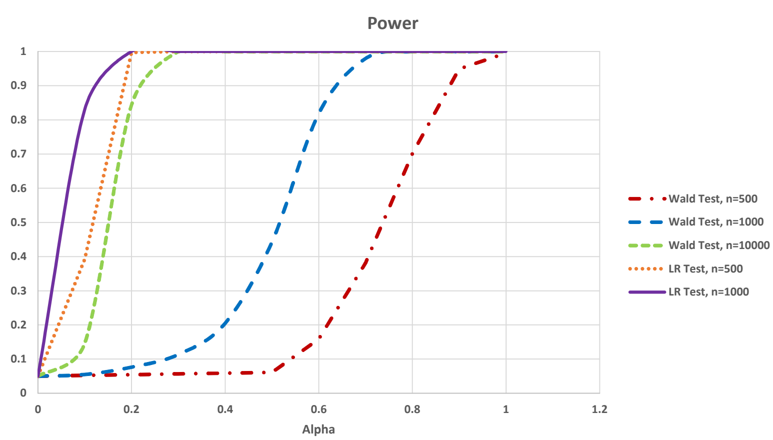

Figure 4: Power

Table 1: Assortative Mating Test Statistics

	log Predicted Wage		Years of Education	
Year	Wald Test	LR test	Wald Test	LR test
1962	525.5	7314.0	2443.6	5642.5
1967	773.3	6298.7	2981.7	5968.1
1970	604.1	5317.9	4453.5	8447.1
1972	497.9	7318.3	3952.1	8404.5
1975	269.1	4329.7	3456.1	7756.3
1978	318.5	2382.0	3857.4	8507.5
1980	672.9	3119.2	5022.2	10139.1
1985	871.8	3101.7	5121.0	9977.7
1990	564.9	5818.5	6061.8	10134.3
1995	451.1	5098.8	4344.7	8250.6
2000	544.9	5764.1	3799.6	7642.9
2005	631.3	8087.4	5372.7	11930.9
2007	654.3	7502.8	5108.6	11536.9
2010	807.4	7046.4	4525.0	10983.2
2014	1004.0	8168.8	3654.2	10317.1
2015	935.5	7930.8	3946.9	10395.8
2018	1043.6	5283.8	2964.6	8854.5
2019	1183.9	5208.0	2897.4	8604.3
# Cells	10		5	
DF	81	99	16	24

amount of assortative mating by level of education. Among other analyses, they use the pseudo-Wald test and the LR test presented in this paper to test for the existence of positive assortative mating. Test results are reported in Table 1. As can be seen in the table, the LR test statistics are an order of magnitude larger than the pseudo-Wald test statistics, both for log predicted wages and years of education. However, for all four tests, the null hypothesis of no assortative mating is rejected at any reasonable significance level for every year of CPS data in favor of the existence of positive assortative mating.

4 Conclusion

[Jay: write first draft.]

References

[1] Carroll, Meg, Hyunjae Kang, and Steven Stern (2021). "The Relationship Between Spouses' Wages Over Time." Unpublished manuscript.

[2] Chiappori, Pierre-André, Murat Iyigun, and Yoram Weiss (2009). "Investment in Schooling and the Marriage Market." *American Economic Review.* 99(5): 1689-1713.

[3] Chiappori, Pierre-André and Bernard Salanié (2016). "The Econometrics of Matching Models." *Journal of Economic Literature.* 54(3): 832-861.

[4] Chiappori, Pierre-André, Bernard Salanié, and Yoram Weiss (2011). "Partner Choice and the Marital College Premium." *American Economic Review.* 107(8): 2109-2167.

[5] Choo, Eugene and Aloysius Siow (2006a). "Estimating a Marriage Matching Model with Spillover Effects." *Demography.* 43(3): 463–490.

[6] Choo, Eugene and Aloysius Siow (2006b). "Who Marries Whom and Why." *Journal of Political Economy.* 114(1): 175–201.

[7] Cragg, John and Stephen Donald (1996). "On the Asymptotic Properties of LDU-Based Tests of the Rank of a Matrix." *Journal of the American Statistical Association.* 91(435): 1301-1309.

[8] Cragg, John and Stephen Donald (1997). "Inferring the Rank of a Matrix." *Journal of Econometrics.* 76(1-2): 223-250.

[9] Dagsvik, John (1994). "Discrete and Continuous Choice, Max-Stable Processes, and Independence from Irrelevant Attributes." *Econometrica.* 62(5): 1179-1205.

[10] Dagsvik, John (2000). "Aggregation in Matching Markets." *International Economic Review.* 41(1): 27-58.

[11] Dupuy, Arnaud and Alfred Galichon (2014). "Personality Traits and the Marriage Market." *Journal of Political Economy.* 122(6): 1271–1319.

[12] Fox, Jeremy (2010). "Estimating the Employer Switching Costs and Wage Responses of Forward-Looking Engineers." *Journal of Labor Economics.* 28(2): 357-412.

[13] Fox, Jeremy (2018). "Estimating Matching Games with Transfers." *Quantitative Economics.* 9: 1–38.

[14] Fox, Jeremy, Chenyu Yang, and David Hsu (2018). "Unobserved Heterogeneity in Matching Games." *Journal of Political Economy.* 126(4): 1339-1373.

[15] Galichon, Alfred and Bernard Salanié (2021). "Cupid's Invisible Hand: Social Surplus and Identification in Matching Models." *Review of Economic Studies.* Forthcoming.

[16] Gill, Len and Arthur Lewbel (1992). "Testing the Rank and Definiteness of Estimated Matrices With Applications to Factor, State-Space and ARMA Models." *Journal of the American Statistical Association.* 87(419): 766-776.

[17] Gourieroux, Christian, Alberto Holly, and Alain Monfort (1982). "Likelihood Ratio Test, Wald Test, and Kuhn-Tucker Test in Linear Models with Inequality Constraints on the Regression Parameters." *Econometrica*. 50(1): 63-80.

[18] Heckman, James (1979). "Sample Selection Bias as a Specification Error." *Econometrica*. 47(1): 153-161.

[19] Kodde, David and Franz Palm (1986). "Wald Criteria for Jointly Testing Equality and Inequality Restrictions." *Econometrica*. 54(5): 1243-1248.

[20] Kudo, Akio (1962). "A Multivariate Analogue of the One-Sided Test." *Biometrika*. 50(3): 403-418.

[21] Kwiatkowski, Denis, Peter C.B. Phillips, Peter Schmidt, and Yeongchol Shin (1992). "Testing the Null of Stationarity Against the Alternative of a Unit Root: How Sure Are We That Economic Time Series Have a Unit Root?" *Journal of Econometrics*. 54(1-3): 159–178.

[22] Leybourne, Stephen and Brenden McCabe (1994). "A Consistent Test for a Unit Root." *Journal of Business and Economic Statistics*. 12(2): 157-166.

[23] Mourifié, Ismael and Aloysius Siow (2021). "The Cobb-Douglas Marriage Matching Function: Marriage Matching with Peer and Scale Effects." *Journal of Labor Economics*. 39(S1): S239 - S274.

[24] Shaked, Moshe and J. George Shanthikumar (2007). *Stochastic Orders*. New York: Springer.

[25] Siow, Aloysius (2015). "Testing Becker's Theory of Positive Assortative Matching." *Journal of Labor Economics*. 33(2): 409-441.

[26] Suen, Wing and Hon-Kwong Lui (1999). "A Direct Test of the Efficient Marriage Market Hypothesis." *Economic Inquiry*. 37(1): 29–46.

[27] Xiao, Zhijie (2001). "Testing the Null Hypothesis of Stationarity Against an Autoregressive Unit Root Alternative." *Journal of Time Series Analysis*. 22: 87-105.

5 Papers to Discuss

References

[1] [Jay] Abowd, John, Francis Kramarz, and David Margolis (1999). "High Wage Workers and High Wage Firms." *Econometrica*. 67(2): 251-334.

[2] [Steve] Anderson, Gordon and Teng Wah Leo (2013). "An Empirical Examination of Matching Theories: The One Child Policy, Partner Choice and Matching Intensity in Urban China." *Journal of Comparative Economics*. 41(2): 468-489.

[3] [Steve] Fernández, Raquel, Nezih Guner, and John Knowles (2005). "Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality." *Quarterly Journal of Economics.* 120(1): 273-344.

[4] [Steve] Graham, Bryan (2011). "Econometric Methods for the Analysis of Assignment Problems in the Presence of Complementarity and Social Spillovers." *Handbook of Social Economics, Vol. 1B.* (eds.) Jess Benhabib, Alberto Bisin, and Matthew Jackson. 965–1052. Amsterdam: North-Holland.

[5] [Jay] Lentz, Rasmus and Jesper Bagger (2009). "An Empirical Model of Wage Dispersion with Sorting." 2009 Meeting Papers 964, Society for Economic Dynamics.

[6] [Jay] Lise, Jeremy, Costas Meghir, and Jean-Marc Robin (2016). "Matching, Sorting and Wages." *Review of Economic Dynamics.* 19: 63-87.

[7] [Jay] Liu, Haoming and Jingfeng Lu (2006). "Measuring the Degree of Assortative Mating." *Economics Letters.* 92(3): 317-322.

[8] [Jay] Mendes, Rute, Gerard van den Berg, and Maarten Lindeboom (2010). "An Empirical Assessment of Assortative Matching in the Labor Market." *Labour Economics.* 17(6): 919-929.

[9] [Steve] Wang, Yazhen (1996). "A Likelihood Ratio Test against Stochastic Ordering in Several Populations." *Journal of the American Statistical Association.* 91: 1676-1683.